Problem 1.
Below are transactional memory executions. For each execution:

- Specify whether it is opaque or not.
- If it is not, suggest a modification to make it opaque.
- Specify an equivalent sequential execution of transactions.

Reminder: An execution is opaque if it is equivalent to some sequential execution in which every transaction, even aborted or unfinished, observes a consistent state of the memory. Transaction $T$ in a sequential execution observes a consistent state of the memory if for every transactional variable $x$ every read operation on $x$ within the transaction returns: (I) the value written by the last write operation on $x$ in the transaction $T$, or (II) the value written by the last write operation on $x$ within a committing transaction, if there are no write operations on $x$ within the execution, or (III) the initial value of $x$ if there are no write operations on $x$ within the execution.

---

**Figure 1:** Transactional execution 1.

```
p_1  T_1: \begin{align*}
x &.write(1) \\
y &.write(2) \\
\end{align*} \quad \bullet \text{commit}
p_2  T_2: \begin{align*}
x &.read \rightarrow 0 \\
y &.read \rightarrow 0 \\
\end{align*} \quad \bullet \text{commit}
```

**Figure 2:** Transactional execution 2.

```
p_1  T_1: \begin{align*}
x &.write(1) \\
y &.write(2) \\
\end{align*} \quad \bullet \text{commit}
p_2  T_2: \begin{align*}
x &.read \rightarrow 0 \\
y &.read \rightarrow 0 \\
\end{align*} \quad \bullet \text{commit}
```

**Figure 3:** Transactional execution 3.

```
p_1  T_1: \begin{align*}
x &.write(1) \\
y &.write(2) \\
\end{align*} \quad \bullet \text{commit}
p_2  T_2: \begin{align*}
x &.read \rightarrow 0 \\
y &.read \rightarrow 0 \\
\end{align*} \quad \bullet \text{commit}
```
Solution

- Figure 1. Yes. An equivalent serial execution is $T_2 \cdot T_1$.

- Figure 2. Yes. An equivalent serial execution is $T_2 \cdot T_1$.

- Figure 3. Yes. An equivalent serial execution is $T_2 \cdot T_1$.

- Figure 4. No. The execution is not opaque because $T_3$ observes results of $T_1$'s actions even though $T_1$ is aborted. One way to make it opaque is to have the read operations in $T_3$ return 0. In this case an equivalent sequential execution is $T_1 \cdot T_2 \cdot T_3$.

- Figure 5. No. The executions is not opaque because if $T_1$ is serialized before $T_2$, then $T_2$ does not observe the write to $y$; and if $T_2$ is serialized before $T_1$, then $T_1$ does not observe the write to $x$. One way to make the execution opaque is to abort one of the transactions. Another is to have read operation in $T_1$ return 1. In this case an equivalent serial execution is $T_2 \cdot T_1$.

- Figure 6. Yes. An equivalent sequential execution is $T_1 \cdot T_2$. 

Figure 4: Transactional execution 4.

Figure 5: Transactional execution 5.

Figure 6: Transactional execution 6.
Problem 2.
Implement the following objects using transactional memory:

- A snapshot.
- A strong counter.
- A compare-and-swap that works on several locations in an array. It takes the indices of the locations, expected old values and the new values as parameters. Only if all the locations in the array have the expected values, it swaps them with the new ones.
Solution

To implement these objects using transactional memory, we only need to enclose their sequential specification in an atomic block.

Snapshot:
uses: array[M]

upon Snapshot do
    begin_transaction;
    for i = 1 to M do
        ret[i] ← array[i];
    end_transaction;
    return ret

Counter:
initially: count = 0

upon Inc do
    begin_transaction;
    ret ← count;
    count ← count + 1;
    end_transaction;
    return ret

CASN:
uses: array[M]

upon CASN(idx, oldv, newv) do
    begin_transaction;
    L ← length(idx);
    for i = 1 to L do
        if array[idx[i]] ≠ oldv[i] then
            end_transaction;
            return array
        for i = 1 to L do
            array[idx[i]] ← newv[i]
    end_transaction;
    return array