- Shared Memory -

R. Guerraoui

Distributed Programming Laboratory

lpdwww.epfl.ch
The application model

Registers

P1

P2

P3
Register (assumptions)

- For presentation simplicity, we assume registers of integers.
- We also assume that the initial value of a register is 0 and this value is initialized (written()) by some process before the register is used.
- We assume that every value written is uniquely identified (this can be ensured by associating a process id and a timestamp with the value).
Register: specification

- Assume a register that is local to a process, i.e., accessed only by one process:

- In this case, the value returned by a `Read()` is the last value written
Sequential execution

P1 $\rightarrow$ W(5) $\rightarrow$ R() $\rightarrow$ W(6) $\rightarrow$ P2
Sequential execution

P1

W(5)

R() -> 5

P2

W(6)

R() -> 6
Concurrent execution

P1

R_1() -> ?  R_2() -> ?  R_3() -> ?

P2

W(5)  W(6)
Execution with failures

P1

W(5)

P2

W(6) crash

R() -> ?
Regular register

- It assumes only \textit{one} writer;

- It provides \textit{strong} guarantees when there is no concurrent or failed operations (invoked by processes that fail in the middle)

- When some operations are concurrent, or some operation fails, the register provides \textit{minimal} guarantees
Regular register

- **Read()** returns:

  ✓ **the last value** written if there is no concurrent or failed operations

  ✓ and otherwise the last value written or **any** value concurrently written, i.e., the input parameter of some **Write()**
Execution

P1

<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>R1()</td>
<td>R2()</td>
<td>R3()</td>
</tr>
</tbody>
</table>

P2

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>W(5)</td>
<td>W(6)</td>
</tr>
</tbody>
</table>
Results 1

\[
\begin{array}{c}
P1 \\
| \hline
R_1() \rightarrow 5 \hspace{1cm} R_2() \rightarrow 0 \hspace{1cm} R_3() \rightarrow 25 \\
\end{array}
\]

\[
\begin{array}{c}
P2 \\
| \hline
W(5) \hspace{1cm} W(6) \\
\end{array}
\]
Results 2

R_1() -> 5  R_2() -> 6  R_3() -> 5

W(5)  W(6)
Results 3

P1

W(5)

P2

W(6) ~ crash

R() -> 5
Results 4

P1

W(5)

P2

W(6)

R() -> 6

crash
Correctness

- Results 1: non-regular register (safe)
- Results 2; 3; 4: regular register
Regular register algorithms

R. Guerraoui
Overview of this lecture

(1) *Overview of a register algorithm*
(2) *A bogus algorithm*
(3) *A simplistic algorithm*
(4) *A simple fail-stop algorithm*
(5) *A tight asynchronous lower bound*
(6) *A fail-silent algorithm*
A distributed system
Shared memory model

Registers

P1

P3
Message passing model
Implementing a register

- From message passing to shared memory

- Implementing the register comes down to implementing `Read()` and `Write()` operations at every process
Implementing a register

- Before returning a `Read()` value, the process must communicate with other processes.

- Before performing a `Write()`, i.e., returning the corresponding `ok`, the process must communicate with other processes.
Overview of this lecture

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(6) A fail-silent algorithm
A bogus algorithm

- We assume that channels are reliable (perfect point to point links)

- Every process $p_i$ holds a copy of the register value $v_i$
A bogus algorithm

- **Read() at pi**
  - ✓ Return \( v_i \)

- **Write(v) at pi**
  - ✓ \( v_i := v \)
  - ✓ Return ok

- The resulting register is live but not safe:
  - ✓ Even in a sequential and failure-free execution, a **Read()** by pj might not return the last written value, say by pi
No safety

R_1() -> 0  R_2() -> 0  R_3() -> 0

P1

P2

W(5)  W(6)
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A simplistic algorithm

- We still assume that channels are reliable but now we also assume that no process fails.

- Basic idea: one process, say p1, holds the value of the register.
A simplistic algorithm

- Read() at pi
  ✓ send [R] to p1
  ✓ when receive [v]
  ✓ Return v

- Write(v) at pi
  ✓ send [W,v] to p1
  ✓ when receive [ok]
  ✓ Return ok

- At p1:
  T1:
  when receive [R] from pi
  send [v1] to pi
  ✓ Return v

  T2:
  when receive [W,v] from pi
  ✓ v1 := v
  ✓ send [ok] to pi
Correctness (liveness)

- By the assumption that
  - (a) no process fails,
  - (b) channels are reliable

  no wait statement blocks forever, and hence every invocation eventually terminates
Correctness (safety)

(a) If there is no concurrent or failed operation, a `Read()` returns the last value written

(b) A `Read()` must return some value concurrently written or the last value written

NB. If a `Read()` returns a value written by a given `Write()`, and another `Read()` that starts later returns a value written by a different `Write()`, then the second `Write()` cannot start after the first `Write()` terminates
Correctness (safety – 1)

(a) If there is no concurrent or failed operation, a \textit{Read()} returns the last value written

Assume a Write(x) terminates and no other Write() is invoked. The value of the register is hence x at p1. Any subsequent Read() invocation by some process \( p_j \) returns the value of p1, i.e., x, which is the last written value.
(b) A \textit{Read()} returns the previous value written or the value concurrently written

Let $x$ be the value returned by a \textit{Read}(): by the properties of the channels, $x$ is the value of the register at $p1$. This value does necessarily come from a concurrent or from the last \textit{Write}().
What if?

Processes might crash?

If p1 crashes, then the register is not live (wait-free)

If p1 is always up, then the register is regular and wait-free
Overview of this lecture

1. Overview of a register algorithm
2. A bogus algorithm
3. A simplistic algorithm
4. A simple fail-stop algorithm
5. A tight asynchronous lower bound
6. A fail-silent algorithm
A fail-stop algorithm

We assume a *fail-stop* model; more precisely:

- any number of processes can fail by crashing (no recovery)
- channels are reliable
- failure detection is perfect (we have a perfect failure detector)
A fail-stop algorithm

- We implement a *regular* register
- every process pi has a local copy of the register value vi
- every process reads *locally*
- the writer writes *globally*, i.e., at all (non-crashed) processes
A fail-stop algorithm

Write(v) at pi
- send \([W, v]\) to all
- for every pj, wait until either:
  - receive \([ack]\) or
  - suspect [pj]
- Return ok

At pi:
- when receive \([W,v]\) from pj
- \(vi := v\)
- send [ack] to pj

Read() at pi
- Return \(vi\)
Correctness (liveness)

✓ A Read() is local and eventually returns

✓ A Write() eventually returns, by the
  (a) the strong completeness property
  of the failure detector, and
  (b) the reliability of the channels
Correctness (safety – 1)

(a) In the absence of concurrent or failed operation, a Read() returns the last value written

Assume a Write(x) terminates and no other Write() is invoked. By the accuracy property of the failure detector, the value of the register at all processes that did not crash is x. Any subsequent Read() invocation by some process pj returns the value of pj, i.e., x, which is the last written value.
Correctness (safety – 2)

(b) A Read() returns the value concurrently written or the last value written

Let x be the value returned by a Read(): by the properties of the channels, x is the value of the register at some process. This value does necessarily come from the last or a concurrent Write().
What if?

- Failure detection is not perfect

- Can we devise an algorithm that implements a regular register and tolerates an arbitrary number of crash failures?
Overview of this lecture

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6. A fail-silent algorithm
Lower bound

- **Proposition:** any wait-free asynchronous implementation of a regular register requires a majority of correct processes

- Proof (sketch): assume a Write(v) is performed and n/2 processes crash, then a Read() is performed and the other n/2 processes are up: the Read() cannot see the value v

- The impossibility holds even with a 1-1 register (one writer and one reader)
The majority algorithm [ABD95]

- We assume that p1 is the writer and any process can be reader
- We assume that a majority of the processes is correct (the rest can fail by crashing – no recovery)
- We assume that channels are reliable
- Every process $p_i$ maintains a local copy of the register $v_i$, as well as a sequence number $s_{ni}$ and a read timestamp $r_{si}$
- Process $p_1$ maintains in addition a timestamp $t_{s1}$
Algorithm - Write()

- Write(v) at p1
  - ts1++
  - send [W,ts1,v] to all
  - when receive [W,ts1,ack] from majority
  - Return ok

- At pi
  - when receive [W,ts1, v] from p1
  - if ts1 > sni then
    - vi := v
    - sni := ts1
    - send [W,ts1,ack] to p1
Algorithm - Read()

- Read() at pi
  - rsi++
  - send [R,rsi] to all
  - when receive [R,rsj,snj,vj] from majority
  - v := vj with the largest snj
  - Return v

- At pi
  - when receive [R,rsj] from pj
  - send [R,rsj,sni,vi] to pj
What if?

Any process that receives a write message (with a timestamp and a value) updates its value and sequence number, i.e., without checking if it actually has an older sequence number.
Old writes

\[ P1 \quad W(5) \quad W(6) \quad P2 \]

\[ sn1 = 1; v1 = 5 \quad sn1 = 2; v1 = 6 \]

\[ P3 \quad sn2 = 1; v2 = 5 \]

\[ sn3 = 2; v3 = 6 \quad sn3 = 1; v3 = 5 \]

\[ R() \rightarrow 5 \]
Correctness 1

✓ Liveness: Any Read() or Write() eventually returns by the assumption of a majority of correct processes (if a process has a newer timestamp and does not send \([W, ts1, ack]\), then the older Write() has already returned)

✓ Safety 2: By the properties of the channels, any value read is the last value written or the value concurrently written
Correctness 2 (safety – 1)

(a) In the absence of concurrent or failed operation, a Read() returns the last value written.

Assume a Write(x) terminates and no other Write() is invoked. A majority of the processes have x in their local value, and this is associated with the highest timestamp in the system. Any subsequent Read() invocation by some process pj returns x, which is the last written value.
What if?

Multiple processes can write concurrently?
Concurrent writes

P1
\[
\begin{aligned}
W(5) & \quad W(6) \\
\text{ts1} = 1 & \quad \text{ts1} = 2
\end{aligned}
\]

P2
\[
R() \rightarrow 6
\]

P3
\[
\begin{aligned}
W(1) \\
\text{ts3} = 1
\end{aligned}
\]
Overview of this lecture

(1) Overview of a register algorithm
(2) A bogus algorithm
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(5) A tight asynchronous lower bound
(6) A fail-silent algorithm
- Atomic register specification -

R. Guerraoui
Distributed Programming Laboratory
lpdwww.epfl.ch
The application model

Registers

P1

P2

P3
Sequential execution

P1

W(5)

P2

R()

R()

W(6)
Sequential execution

P1

W(5)  R() -> 5

P2

W(6)  R() -> 6
Concurrent execution

P1

\[ \text{R}_1() \rightarrow ? \quad \text{R}_2() \rightarrow ? \quad \text{R}_3() \rightarrow ? \]

P2

\[ \text{W}(5) \quad \text{W}(6) \]
Execution with failures

P1

W(5)

P2

W(6)  crash

R() -> ?
Safety

- *An atomic register* provides strong guarantees even when there is concurrency and failures.

- The execution is equivalent to a sequential and failure-free execution (*linearization*).
Atomic register

- Every failed (write) operation appears to be either complete or not to have been invoked at all

And

- Every complete operation appears to be executed at some instant between its invocation and reply time events
Execution 1

P1: R₁() -> 5
P2: W(5)

P1: R₂() -> 0

P1: R₃() -> 25
P2: W(6)
Execution 2

P1

R₁() -> 5

P2

W(5)

R₂() -> 6

R₃() -> 5

W(6)
Execution 3

P2

P1

W(5)  R1() -> 5  R2() -> 5  R3() -> 5

W(6)
Execution 4

P1

R_1() -> 5

R_2() -> 6

R_3() -> 6

P2

W(5)

W(6)
Execution 5

P1

W(5)

R() -> 5

W(6)

crash

P2
Execution 6

P1

P2

W(5)

R() -> 5

R() -> 6

W(6) crash
Execution 7

P1

W(5)

P2

W(5)  R() -> 6  R() -> 5

W(6)  crash
Correctness

- Execution 1: non-regular (safe)
- Executions 2 and 7: non-atomic (regular)
- Executions 3; 4, 5 and 6: atomic
Regular vs Atomic

- For a regular register to be atomic, two successive `Read()` must not overlap a `Write()`

- The regular register might in this case allow the first `Read()` to obtain the new value and the second `Read()` to obtain the old value
Atomic register algorithms

R. Guerraoui

Distributed Programming Laboratory
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Overview of this lecture

(1) *From regular to atomic*
(2) *A 1-1 atomic fail-stop algorithm*
(3) *A 1-N atomic fail-stop algorithm*
(4) *A N-N atomic fail-stop algorithm*
(5) *From fail-stop to fail-silent*
Fail-stop algorithms

We first assume a fail-stop model; more precisely:

- any number of processes can fail by crashing (no recovery)
- channels are reliable
- failure detection is perfect
The simple algorithm

Consider our fail-stop regular register algorithm:
- Every process has a local copy of the register value.
- Every process reads locally.
- The writer writes globally, i.e., at all (non-crashed) processes.
The simple algorithm

Write(v) at pi
- send [W,v] to all
- for every pj, wait until either:
  - received [ack] or
  - suspected [pj]
- Return ok

At pi:
- when receive [W,v] from pj
  - vi := v
- send [ack] to pj

Read() at pi
- Return vi
Atomicity?

P1

\[ R_1() \rightarrow 5 \]

\[ v1 = 5 \]

P2

\[ W(5) \]

P3

\[ R_3() \rightarrow 5 \]

\[ v3 = 5 \]
Linearization?

R₁() -> 5
R₂() -> 6

R₃() -> 5  ??
Fixing the pb: read-globally

Read() at pi
- send [W,vi] to all
- for every pj, wait until either:
  - receive [ack] or
  - suspect [pj]
- Return vi
Still a problem

R() -> 5

P1

W(5) W(6)

P2

R() -> 5

P3
Linearization?

$R_1() \rightarrow 5$

P1

$W(5)$

P2

$W(6)$

P3

$R_3() \rightarrow 5$
Overview of this lecture

(1) From regular to atomic
(2) A 1-1 atomic fail-stop algorithm
(3) A 1-N atomic fail-stop algorithm
(4) A N-N atomic fail-stop algorithm
(5) From fail-stop to fail-silent
A fail-stop 1-1 atomic algorithm

Write(v) at p1
- send [W,v] to p2
- Wait until either:
  - receive [ack] from p2
  - suspect [p2]
- Return ok

At p2:
- when receive [W,v] from p1
- v2 := v
- send [ack] to p2

Read() at p2
- Return v2
A fail-stop 1-N algorithm

every process maintains a local value of the register as well as a sequence number

the writer, p1, maintains, in addition a timestamp ts1

any process can read in the register
A fail-stop 1-N algorithm

Write(v) at p1
- ts1++
- send [W,ts1,v] to all
- for every pi, wait until either:
  - receive [ack] or
  - suspect [pi]
- Return ok

Read() at pi
- send [W,sni,vi] to all
- for every pj, wait until either:
  - receive [ack] or
  - suspect [pj]
- Return vi
A 1-N algorithm (cont’d)

At pi

When pi receive \([W,ts,v]\) from pj
if ts > sni then
  \(v_i := v\)
  \(sni := ts\)
send \([\text{ack}]\) to pj
Why not N-N?

P1

P2

\(W(X)\) \(\rightarrow\) \(W(Y)\)

R() \(\rightarrow\) Y

P3

\(W(Z)\)
The Write() algorithm

- **Write(v) at pi**
  - ✓ send [W] to all
  - ✓ for every pj wait until
    - ● receive [W,snj] or
    - ● suspect pj
  - ✓ (sn,id) := (highest snj + 1,i)
  - ✓ send [W,(sn,id),v] to all
  - ✓ for every pj wait until
    - ● receive [W,(sn,id),ack] or
    - ● suspect pj
  - ✓ Return ok

- **At pi**
  - **T1:**
    - ✓ when receive [W] from pj
      - ● send [W,sn] to pj
  - **T2:**
    - ✓ when receive [W,(snj,idj),v] from pj
    - ✓ If (snj,idj) > (sn,id) then
      - ● vi := v
      - ● (sn,id) := (snj,idj)
    - ✓ send [W,(snj,idj),ack] to pj
The Read() algorithm

- **Read() at pi**
  - ✓ send [R] to all
  - ✓ for every pj wait until
    - ● receive [R,(snj,idj),vj] or
    - ● suspect pj
  - ✓ v = vj with the highest (snj,idj)
  - ✓ (sn,id) = highest (snj,idj)
  - ✓ send [W,(sn,id),v] to all
  - ✓ for every pj wait until
    - ● receive [W,(snj,id),ack] or
    - ● suspect pj
  - ✓ Return v

- **At pi**
  - **T1:**
    - ✓ when receive [R] from pj
      - ● send [R,(snj,id),vi] to pj
  - **T2:**
    - ✓ when receive [W,(snj,idj),v] from pj
    - ✓ if (snj,idj) > (snj,id) then
      - ● vi := v
      - ● (snj,idj) := (snj,idj)
    - ✓ send [W,(snj,idj),ack] to pj
Overview of this lecture

(1) From regular to atomic
(2) A 1-1 atomic fail-stop algorithm
(3) A 1-N atomic fail-stop algorithm
(4) A N-N atomic fail-stop algorithm
(5) From fail-stop to fail-silent
From fail-stop to fail-silent

- We assume a majority of correct processes

- In the 1-N algorithm, the writer writes in a majority using a timestamp determined locally and the reader selects a value from a majority and then imposes this value on a majority

- In the N-N algorithm, the writers determines first the timestamp using a majority