

# Registers

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# *Register*

- A *register* has two operations: *read()* and *write()*
- Sequential specification
- • *read()*
  - return(x)
- • *write(v)*
  - $x \leftarrow v$ ; return(ok)

# Space of registers

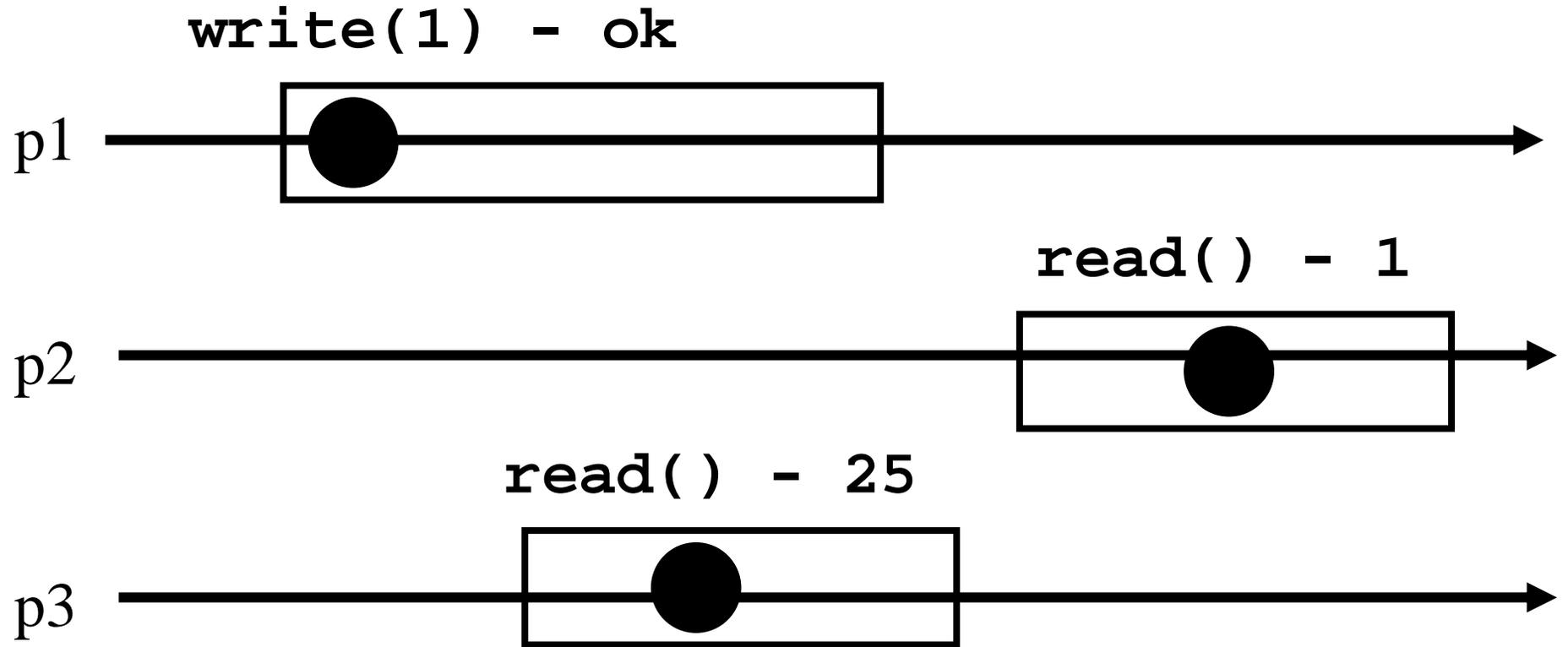
- ☛ Dimension 1: binary (boolean) – multivalued
- ☛ Dimension 2: safe – regular – atomic
- ☛ Dimension 3: SRSW – MRSW – MRMW

# Space of registers

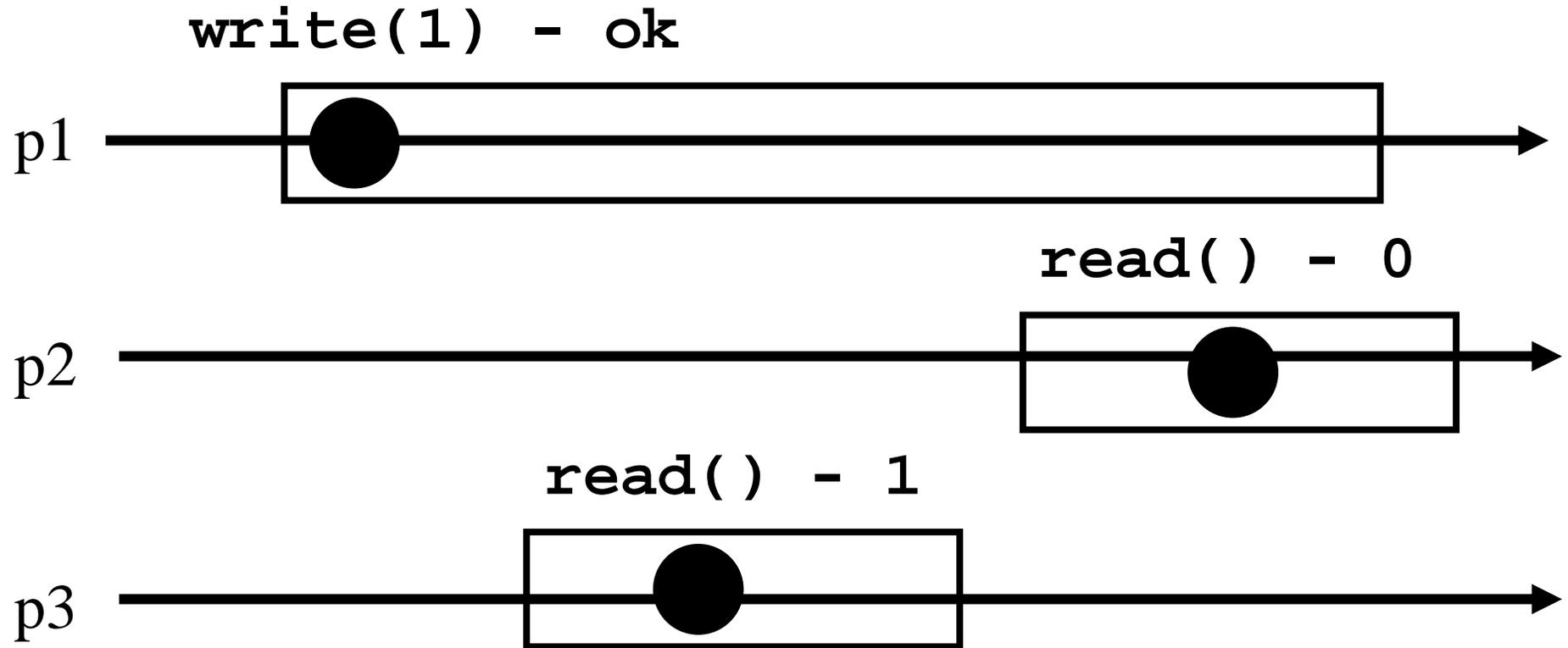
- Theorem: A multivalued MRMW atomic *register* can be implemented with binary SRSW safe *register*

(2 decades of research in distributed computing)

# Safe execution



# Regular execution



# Simplifications

- We assume that *registers* contain only integers
- Unless explicitly stated otherwise, *registers* are initially supposed to contain 0
- The process executing the code is implicitly assumed to be  $p_i$
- (we assume a system of  $N$  processes)

# Conventions

- Shared registers are denoted *Reg*
- The operations to be implemented are denoted *Read()* and *Write()*
- Those of the base registers are denoted *read()* and *write()*
- We omit the *return(ok)* instruction at the end of *Write()* implementations

# From (binary) SRSW safe to (binary) MRSW safe

- We use an array of SRSW *registers*

Reg[1,...,N]

- **Read()**

- return (Reg[i].read());

- **Write(v)**

- for j = 1 to N

- Reg[j].write(v);

# From (binary) SRSW safe to (binary) MRSW safe

- The transformation works also for multi-valued *registers* and regular ones
- It does not however work for atomic *registers*

# From Binary MRSW safe to Binary MRSW regular

- ☞ We use one MRSW safe register
- ☞ **Read()**
  - ☞ `return(Reg.read());`
- **Write(v)**
  - ☞ if `old ≠ v` then
    - ☞ `Reg.write(v);`
    - ☞ `old := v;`

# From Binary MRSW safe to Binary MRSW regular

- The transformation works for single reader *registers*
- It does not work for multi-valued *registers*
- It does not work for atomic *registers*

# From binary to M-Valued MRSW regular

- We use an array of MRSW *registers*  
Reg[0,1,...,M] init to [1,0,...,0]
- **Read()**
  - for j = 0 to M
    - if Reg[j].read() = 1 then return(j)
- **Write(v)**
  - Reg[v].write(1);
  - for j=v-1 downto 0
    - Reg[j].write(0);

# From binary to M-Valued MRSW regular

- The transformation would not work if the Write() would first write 0s and then 1
- The transformation works for *regular* but ***NOT*** for *atomic* registers

# From SRSW regular to SRSW atomic

- We use one SRSW *register* Reg and two local variables t and x
- **Read()**
  - $(t',x') = \text{Reg.read}();$
  - if  $t' > t$  then  $t := t'; x := x';$
  - return(x)
- **Write(v)**
  - $t := t + 1;$
  - $\text{Reg.write}(v,t);$

# From SRSW regular to SRSW atomic

- The transformation would not work for multiple readers
- The transformation would not work without timestamps (variable  $t$  representing logical time)

# From SRSW atomic to MRSW atomic

- We use  $N \times N$  SRSW atomic *registers*  $RReg[(1,1),(1,2),\dots,(k,j),\dots(N,N)]$  to communicate among the readers
  - In  $RReg[(k,j)]$  the reader is  $p_k$  and the writer is  $p_j$
- We also use  $n$  SRSW atomic *registers*  $WReg[1,\dots,N]$  to store new values
  - the writer in all these is  $p_1$
  - the reader in  $WReg[k]$  is  $p_k$

# From SRSW atomic to MRSW atomic (cont'd)

- **Write(v)**
  - $t1 := t1 + 1;$
  - for  $j = 1$  to  $N$ 
    - $WReg.write(v, t1);$

# From SRSW atomic to MRSW atomic (cont'd)

## Read()

- for  $j = 1$  to  $N$  do
  - $(t[j], x[j]) = \text{RReg}[i, j].\text{read}();$
- $(t[0], x[0]) = \text{WReg}[i].\text{read}();$
- $(t, x) := \text{highest}(t[..], x[..]);$
- for  $j = 1$  to  $N$  do
  - $\text{RReg}[j, i].\text{write}(t, x);$
- return( $x$ )

# From SRSW atomic to MRSW atomic (cont'd)

- ☛ The transformation would not work for multiple writers
- ☛ The transformation would not work if the readers do not communicate (i.e., if a reader does not write)

# From MRSW atomic to MRMW atomic

- We use  $N$  MRSW atomic *registers*  $\text{Reg}[1, \dots, N]$ ; the writer of  $\text{Reg}[j]$  is  $p_j$
- **Write( $v$ )**
  - for  $j = 1$  to  $N$  do
    - $(t[j], x[j]) = \text{Reg}[j].\text{read}();$
    - $(t, x) := \text{highest}(t[..], x[..]);$
    - $t := t + 1;$
    - $\text{Reg}[i].\text{write}(t, v);$

# From MRSW atomic to MRMW atomic (cont'd)

## • Read()

- for  $j = 1$  to  $N$  do
  - $(t[j], x[j]) = \text{Reg}[j].\text{read}();$
- $(t, x) := \text{highest}(t[..], x[..]);$
- return( $x$ )