Exercice 1:

N.B: to avoid any confusion, replace "processor i may have failed or not" by "nothing can be said about processor i" (otherwise you can troll with "may or may not have is always true" :))

a. False: some processors can fail because of another reason that the failure of i. Nothing can be said about processor i.
b. True. (see a)
c. False, for the same reason.
d. False. If no processor j≠i fails, this implies "some processor j≠i does not fail" which is the negation of "all processors j≠i fail" which, by contraposition (https://www.youtube.com/watch?v=ik2x_hOEXDw), implies the negation of "processor i fails" which is "processor i does not fail"
e. False
f. True (this implies the only correct formulation of the contraposition)
g. False. Even if the failure of processor i is a cause of the failure of all processors j≠i, it is not **necessarily** the only cause (if ≠ if and only if).
h. True. Nothing can be said about i.
i: False
j: False: "if some processor j≠i does not fail" is the beginning of the contraposition(https://www.youtube.com/watch?v=ik2x_hOEXDw) of statement #, which implies that "processor i has not failed"
k: False
l: True, this is the only correct formulation of the contraposition.

As the only correct formulation of the contraposition, (l) not only implies, but is equivalent to #.

Correct answers are (l) and (f).

Exercice 2:

a. False for the same reason as ex1
b. True for the same reason as ex1
c. False for the same reason as ex1
d. Now this becomes True because of the "eventually".
e. False.
f. This becomes false because of the "eventually".
g. False (other cause possible)
h. True same reason as Ex1
i: False idem
j: True, nothing can be said about processor i because of the "eventually".
k: False
l: False, nothing can be said about i because of the "eventually"

None of them implies #

Exercice 3:

Here is a sketch of a (more than) acceptable prove for the exam:

Let P(n) be proposition "the number of edges in a complete graph of n vertices is n(n-1)/2"

We prove by induction that P(n) is valid for every integer n≥1:

Base case:

for n=1, the number of edges is equal to 0, which corresponds to "P(1) is true".

Induction:

assume that P(n) is true for a certain n≥1,

Let G be an n+1 complete graph
Let v be any vertex in G, and G'=G-{v}, you are left with 1 vertex (v) and an n-vertices complete graph G', the number of edges incoming to v is n (one edge to all the n vertices of G', since G is complete) , and by the induction hypothesis, since G' is an n-vertices complete graph, it contains n(n-1)/2 vertices

leading to a total of n + n(n-1)/ 2 = (n+1)n/2 , which proves P(n+1)

Since P(n) implies P(n+1) and P(1) is true, P(n) is true for every integer n≥1.