Distributed Algorithms

Logic 101
1st exercise session

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Example 1 (Conditional statements)

Write the converse, contrapositive and inverse of the following sentence:

“If process x fails, then process y never receives message m”

Reminder:

Let P be the proposition $p \rightarrow q$:

- The *converse* of P is: $q \rightarrow p$:
- The inverse of P is: $\neg p \rightarrow \neg q$
- The contrapositive of P is: $\neg q \rightarrow \neg p$

Notes:

- Only the contrapositive of a conditional statement is equivalent to it.
- The proposition “p iff q” means that both P and the converse of P are true.

Exercise 1 (Conditional statements)

Write the negation of the following sentence:

“If process x fails, then process y never receives message m”

Reminder:

Let P be a proposition. The negation of P is ¬P (“not P”). For example:

- ¬“7 is odd” = “7 is not odd” = “7 is even” (if you prove it!)
- ¬“All cats are animals” = “Some cats are not animals”

Hints:

- The negation of ¬p → ¬q is *not* p → ¬q.
- Express the implication in terms of *and* and *or* expressions.

Example 2

If the following statement is true:

If process i fails, then instantly all processes j≠i fail

Which of the following are also true?

1. If a process j≠i fails, then process i has failed,
2. If a process j≠i fails, nothing can be said about process i,
3. If a process j≠i fails, then process i has not failed

(continues on next slide)

Example 2 (contd)

4. If no process j≠i fails, nothing can be said about process i,
5. If no process j≠i fails, then process i has failed,
6. If no process j≠i fails, then process i has not failed,
7. If all processes j≠i fail, then process i has failed,
8. If all processes j≠i fail, nothing can be said about process i,
9. If all processes j≠i fail, then process i has not failed,
10. If some process j≠i does not fail, nothing can be said about process i,
11. If some process j≠i does not fail, then process i has failed,
12. If some process j≠i does not fail, then process i has not failed.

Exercise 2

Replace “instantly” with “eventually” in Example 2.

Example 3 (Proof by cases)

Let x, y, z, q be natural numbers such that

$$x^2 + 5y^2 + 5z^2 = q^2$$

Prove that q is even if and only if all of x, y, and z are even as well.

Exercise 3 (Proof by cases)

Prove that $x + |x - 7| \geq 7$

Example 4 (Proof by contradiction)

Prove that the set of prime numbers is infinite.

Exercise 4 (Proof by contradiction)

Prove that if α^2 is even, α is even.

Bonus Exercise 4 (Proof by contradiction)

Prove that √2 is irrational.

Hint: Use the result of exercise 4

Proof by contradiction:

- In order to prove *p*, find a contradiction *q* such that $\neg p \to q$ is true.
- A contradiction always has the form: $q \equiv r \land \neg r$.

Hints:

- Use the result of Exercise 3.
- A rational number is always in the form r/q, where r is integer, q is natural, and r and q *have no common divisor*.

Example 5 (proof by induction)

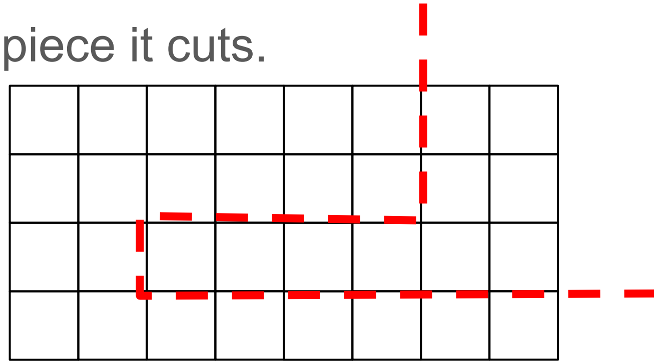
Let a swiss chocolate of rectangular shape *mxn*. What is the smallest number of *cuts* we need to do, in order to break up the chocolate in individual pieces of size 1x1?

A *cut* is defined as any line that:

1. Does not cross itself,
2. Starts and ends on the perimeter of the chocolate piece it cuts.

Note: You cannot consider a cut on two pieces as a single cut, just because these pieces are next to each other.

e.g.

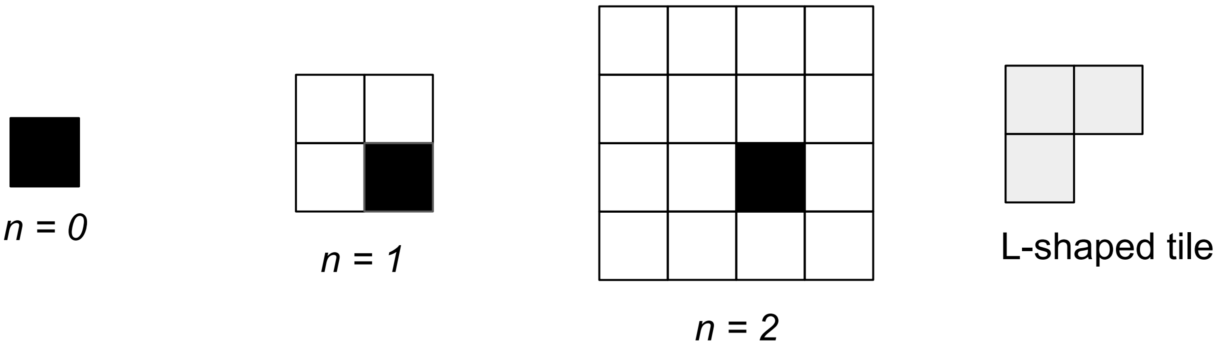


Exercise 5 (proof by induction)

A chessboard of size $2^n x 2^n$ ($n \geq 0$) has all of its squares painted white, except for one arbitrary square, which is painted black.

Prove that for every $n \geq 0$, you can cover all the white squares of the chessboard with L-shaped non-overlapping tiles.

e.g.



Bonus Exercise 5 (proof by induction)

Consider a country with $n \geq 2$ cities. For every pair of different cities x, y, there exists a direct route (single direction) either from x to y or from y to x. Show that there exists a city that we can reach from every other city either directly or through exactly one intermediate city.